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Induced stochastic resonance near a subcritical bifurcation

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The effects of variations in potential barrier height on stochastic resonance in a multistable system are investigated in this experimental work. It is found that small reductions in the potential barrier height tend to shift the stochastic resonance peak to lower values of input noise intensity. For demonstration purposes the work relies on the destabilizing shift produced by near-resonant perturbations on bistable subcritical bifurcations. It is proposed that multistable systems may take advantage of induced stochastic resonance in order to improve the signal-to-noise ratio, especially in cases where only a limited or fixed amount of input noise is available.

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Stochastic resonance (SR) is a phenomenon occurring in nonlinear systems in which there is a cooperative effect between noise and periodic driving. The primary signature of SR is that the addition of noise can improve the signal-to-noise ratio of the output of a periodically modulated multistable system, relative to that observed with no externally injected noise. The theory of SR is well documented [1–4] and the phenomenon has been experimentally studied in several systems [5–7]. In the majority of the work reported thus far, both theoretical as well as experimental, a system with a certain potential function having a *fixed* potential well barrier height and symmetry is usually assumed. Such a system is subjected to a small modulation signal and an improvement in the output signal-to-noise ratio at the modulation frequency is observed by adding noise to the system. In this work, we investigate the effects of *variations* in the effective potential well barrier height on the output signal-to-noise ratio. In certain cases, the externally injected noise intensity is kept fixed while the potential well barrier height is varied, resulting in an improved signal-to-noise ratio. In general, it is found that very small changes in the potential well barrier height and symmetry tend to have profound effects on the output signal-to-noise ratio. The results could have important ramifications in understanding how a multistable system [e.g., optical bistable switches, sensory neurons, biological ion channels, superconducting quantum interference device (SQUID) magnetometers, etc.] may improve its signal-to-noise ratio by slightly varying the associated potential barrier height and symmetry, especially when only a limited amount of input noise is available. The work is primarily experimental in nature and relies on a bistable, hysteretic subcritical bifurcation observed in the nonlinear strain response of a driven magnetostrictive ribbon and on the phenomena of near resonant perturbations [8] which tend to shift the bifurcation point, effectively changing the potential well barrier height and symmetry. In other words, we investigate the effects of near-resonant perturbations on SR near a subcritical bifurcation.

A subcritical bifurcation can be qualitatively understood by the following normal form:

$$\dot{x}=\mu x+ax^3-x^5. \tag{1}$$

Such subcritical bifurcations are considered generic models for studying optical bistability, sudden spatiotemporal pattern changes, Josephson junction dynamics, etc. In Eq. (1) the bifurcation occurs at $\mu=0$. The steady state solution $x_0=0$ is stable for $\mu<0$ and becomes unstable for $\mu>0$. The two stable states for $\mu>0$ are given by

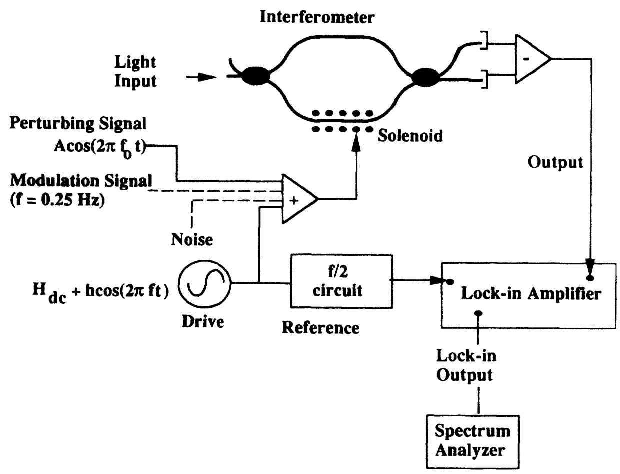

FIG. 1. Schematic of the experimental arrangement.

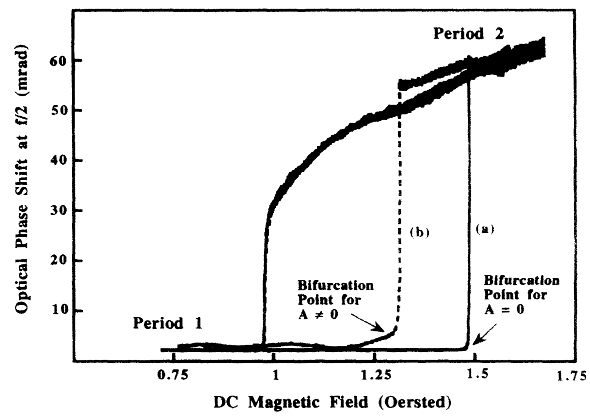

FIG. 2. Experimental data showing induced subcritical bifurcation by near resonant perturbation: (a) subcritical bifurcation curve corresponding to the unperturbed case ($A=0$) and (b) $A=0.0075$ Oe.

$$x_{\pm}=\pm\sqrt{\frac{a}{2}+\frac{1}{2}[(a^2+4\mu)]^{1/2}}. \tag{2}$$

Qualitatively, the subcritical bifurcation can be viewed as follows. If the system is initially on the x_0 branch ($\mu\leqslant 0$) and the bifurcation parameter μ is increased, the system approaches the bifurcation point ($\mu=0$) where it can "suddenly" evolve to the upper branch for $\mu\geqslant 0$. As μ is decreased the system displays a hysteresis. The subcritical bifurcation diagram, associated potential function, and the modulation of the potential wells as a function of μ are well depicted by Colet, De Pasquale, and San Miguel [9]. Such a subcritical bifurcation has been observed in the nonlinear strain response of a driven magnetostrictive ribbon which has been used as a generic two state system to observe SR [10,11]. More recently, the effects of near-resonant perturbation on such period doubling subcritical bifurcations have been investigated [12,13]. Normal form analysis has shown and experiments [13] have confirmed that a near-resonant perturbation tends to shift the bifurcation point in such a way as to induce the subcritical bifurcation. In this work we have exploited the effects of near-resonant perturbations on a subcritical bifurcation to observe a shift in SR peak. The shift is found to be dependent on the strength and detuning of the perturbation.

The experiment involved measuring the dynamic strain response of a magnetically driven $Fe_{78}B_{13}S_9$ amorphous magnetostrictive ribbon (Metglas 2605S-2) using a fiber-optic Mach-Zehnder interferometer (Fig. 1). A small portion (<5 mm) of the ribbon (50 mm×12 mm×25 μm) was bonded to the optical fiber comprising one arm of the interferometer. The phase shift of light propagating in the fiber attached to the ribbon is a direct measure of strain in the ribbon. The interferometer was contained in a solenoid which was driven by a two channel frequency synthesizer (HP3326A), providing a longitudinal magnetic field $H=H_{dc}+h\cos(2\pi ft)$, where H_{dc} is the applied dc field and h is the amplitude of the sinusoidally varying pump field. The perturbing signal, $A\cos(2\pi f_0 t)$, where A and f_0 are the amplitude and the frequency respectively of the perturbing signal, was added with the second channel of the synthesizer. The modulation test signal $[h_{\rm mod}\cos(2\pi f_{\rm mod}t)]$ and band limited (0–100 Hz) Gaussian noise $[\langle\xi(t)\rangle]$ are added similarly. The power spectral density of the strain response output was measured with a dynamic signal analyzer (HP3562A) and the amplitudes of the applied magnetic fields were obtained by measuring the voltage drop across a 1 Ω resistor in series with the solenoid. The experimental arrangement has been described in greater detail in previous work [14]. In order to observe SR, the system was biased such that the wells were symmetric and the low frequency ($f_{\rm mod}=0.25$ Hz) test signal was such that $h_{\rm mod}\leqslant 30\%$ of the hysteresis loop width. The biasing and the modulation conditions ensured that the system did not switch deterministically between the two states (x_0 and $x_{\pm}$).

Figure 2 shows clearly that a near-resonant perturbation induces the onset of a subcritical bifurcation. For the unperturbed case ($A=0$) the magnetostrictive oscillator displayed a distinct period doubling bifurcation for $h=0.15$ Oe with $f=9.6$ kHz and $H_{dc}=1.5$ Oe. The strain response at $f/2$ was detected with a lock-in amplifier (LIA) and plotted as a function of the bifurcation parameter (H_{dc}). The introduction of a perturbing periodic signal ($A>0$) was found to shift the bifurcation point in such a way as to *induce* the subcritical bifurcation. For detuning $\Delta=|(f/2)-f_0|=0.1$ Hz, Fig. 2 shows the measured strain response at $f/2$ as a function of the bifurcation parameter H_{dc} for $A=0$ and 0.0075 Oe. Since it is obvious from the data of Fig. 2 that near-resonant perturbations tend to "narrow" the hysteresis loop it would appear that this effect should also shift the SR curve.

We carried out a series of experiments in which we measured SR near a subcritical bifurcation with and without a near-resonant perturbation. The data in Fig. 3 depict SR curves for $A=0$ and 0.0075 Oe. It is clear that the near-resonant perturbation tends to shift the SR peak to lower values of input noise intensity. This is expected simply due to the fact that near-resonant perturbations tend to shift the bifurcation point such that it narrows the hysteresis loop width associated with a subcritical bifurcation. From the point of view of the potential function associated with the bifurcation, it can be interpreted that the shift in the bifurcation point has the same effect as lowering the potential well

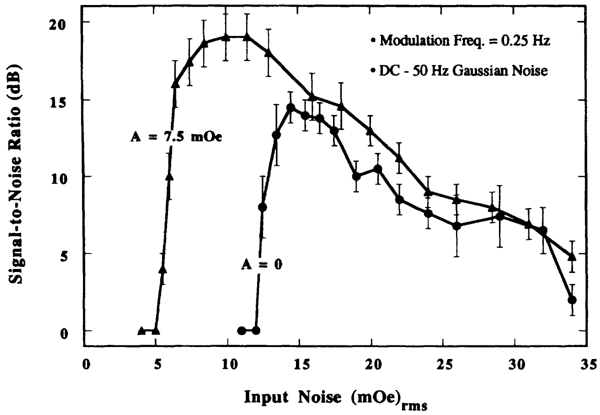

FIG. 3. Data depicting stochastic resonance for the case with (a) $A=0$ and (b) $A=0.0075$ Oe. It is clear that a destabilizing shift of the subcritical bifurcation point, which effectively reduces the potential barrier height, shifts the SR peak to lower values of input noise intensity.

barrier height. Due to a lower potential barrier height the noise intensity required to observe cooperation between the modulation signal and the noise is lower, resulting in the shift of the SR peak. The shift in the SR peak is also accompanied by an overall gain in the signal-to-noise ratio. The signal-to-noise ratio for $A=0.0075$ Oe is larger by about 5 dB with respect to the signal-to-noise ratio for $A=0$. We believe this is due to the shift in the bifurcation point which results not only in lowering the potential barrier height but also changes the symmetry of the potential wells thereby affecting the overall signal-to-noise ratio.

We also performed a set of experiments in which the input noise intensity was kept fixed and the near-resonant perturbation strength and detuning were varied. The modulation signal strength was as stated above and the input noise intensity was such that the output signal-to-noise ratio was very nearly zero [Fig. 4(a)]. In other words, for the given conditions no cooperative effects were occurring between the modulation signal and the noise which could result in appreciable signal-to-noise ratio. However, as the near-resonant perturbation was turned on (i.e., for $A \neq 0$) interwell switching events at the modulation frequency were observed, leading to a substantial signal-to-noise ratio [Fig. 4(b)]. The power spectra of Fig. 4 clearly show that by changing the barrier height, in this case lowering it, a multistable system could improve its signal-to-noise ratio even when a limited or fixed amount of noise is available to it. The output signal-to-noise ratio as a function of the perturbation signal strength and detuning are summarized in Fig. 5. The signal-to-noise ratio tends to saturate for large perturbations and is also strongly dependent on the detuning frequency. This is expected from normal form analysis [13].

The experimentally demonstrated effects of changes in the potential well barrier height and symmetry on SR in multistable systems, especially when limited or fixed noise input intensity is available to it, could have important ramifications. For instance, a multistable system intending to take advantage of SR to improve its signal-to-noise ratio may have access to only a limited amount of noise. Because the system may be unable to increase the modulation signal strength or the noise intensity, especially if they are both external parameters, it will not be able to utilize the SR effect to any benefit. However, by making small changes (<15%) to its potential well barrier height, an internal system parameter, the system could use the available (limited) noise intensity to observe SR and improve its signal-to-noise

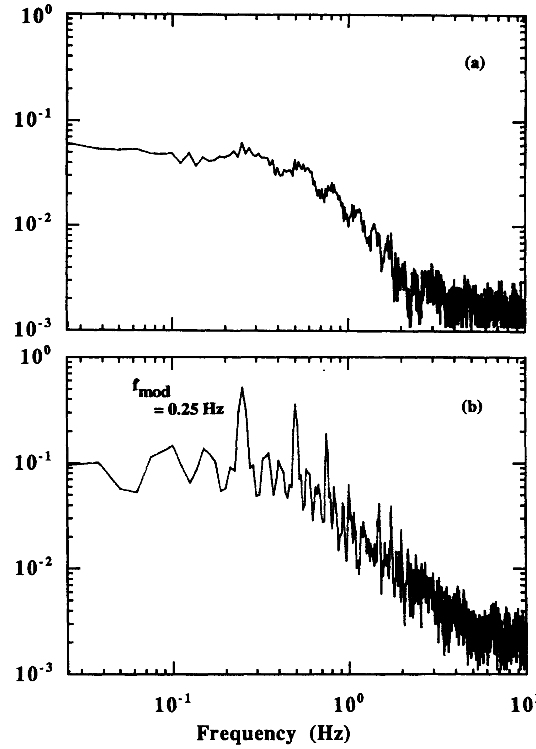

FIG. 4. Power spectra showing the signal-to-noise ratio of the output for a fixed input noise strength. (a) Since the noise intensity is not large enough, no signal-to-noise ratio is observed for the case where $A=0$. (b) By keeping the input noise intensity fixed but introducing a perturbation, $A=0.0075$ Oe, the potential barrier height is lowered enough to provide a substantial signal-to-noise ratio. Notice the Lorentzian noise rolloff.

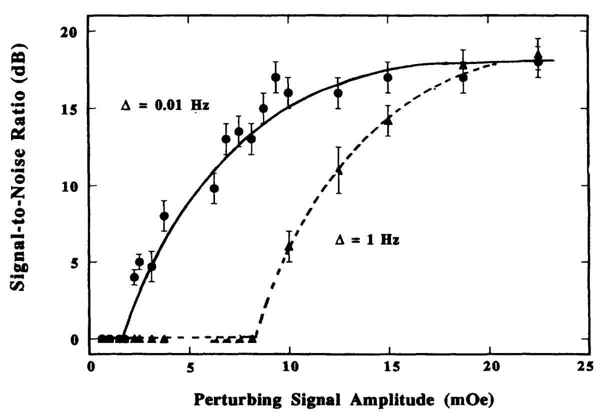

FIG. 5. Dependence of output signal-to-noise ratio on the strength of the perturbing signal amplitude for two different detuning frequencies. The input noise amplitude was held constant at 5 mOe_{rms} which ensured that for $A=0$ the system output showed zero signal-to-noise ratio.

ratio. In the experiment discussed above, this effect was demonstrated in SR near a subcritical bifurcation where a near-resonant perturbation was used to shift the subcritical bifurcation point which effectively altered the potential barrier height. A given system may have its own way to alter its barrier height and it is not suggested that multistable systems must use near-resonant perturbations to vary their barrier height. It is also not suggested that this method is only beneficial for improving the signal-to-noise ratio of the system. It is likely that multistable systems which use noise to transfer information from one state to the other may be able to take advantage of this effect as well.

In conclusion, we have demonstrated dramatic effects of variations in the potential well barrier height on SR in multistable systems. The study was carried out by observing the effects of near-resonant perturbation on SR near a subcritical bifurcation. It is likely that multistable systems relying on the SR effect to improve their signal-to-noise ratio or improve information transfer may be able to use this phenomenon to some advantage, especially in cases where limited amount of noise is available.

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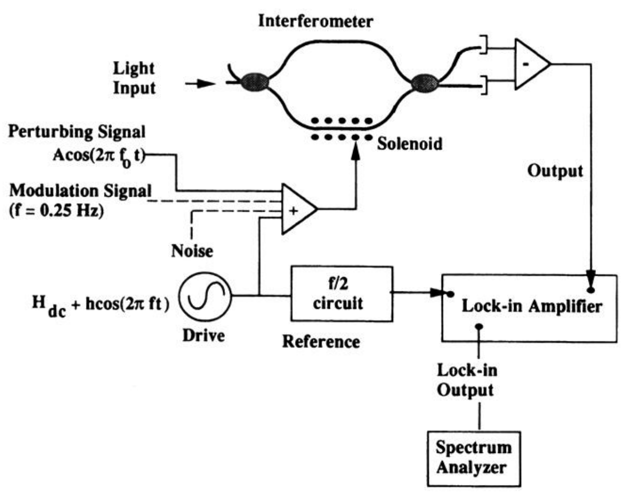

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